215a Final exercises, due on or before Thurs Dec. 10, 3pm

1. Consider the Lagrangian

$$\mathcal{L} = \sum_{f=1}^{4} \bar{\psi}_f i\partial\!\!\!/ \psi_f - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \tfrac{1}{2}m^2 A_\mu A^\mu + \mathcal{L}_{int},$$

$$\mathcal{L}_{int} = -g(\bar{\psi}_1 A\!\!\!/ \gamma_5 \psi_2 + \bar{\psi}_3 A\!\!\!/ \gamma_5 \psi_4) + c.c.$$

(where $+c.c.$ means to add the complex conjugate).

a) Draw pictures and give the Feynman rules for all the propagators and vertices.

b) Compute the decay rate Γ, for $A_\mu \to \bar{\psi}_3 + \psi_4$. Average over the initial A_μ polarizations, and sum over the final spin states.

c) Compute the cross section σ for $\psi_1 + \bar{\psi}_2 \to A$.

d) Compute the amplitude for $\psi_1(p,r) + \psi_3(q,s) \to \psi_2(p',r') + \psi_4(q',s')$.

e) Compute the center of momentum differential cross section for scattering $\psi_1 + \psi_3 \to \psi_2 + \psi_4$, summing over the final spins and averaging over the initial spins.

f) Compute the amplitude for $A_\mu(p,\epsilon_r) + A_\nu(q,\epsilon_s) \to \psi_1(p',r') + \bar{\psi}_1(q',s')$ (compute it in terms of the initial polarizations, and final spins). Does the amplitude vanish if we take the polarization of a vector field to be proportional to its 4-momentum, $\epsilon_r^\mu = p^\mu$? Why?

g) Compute the center of momentum differential cross section for the process of part (f), now summing over final states and averaging over initial ones.

2. Consider the Lagrangian

$$\mathcal{L} = (\partial_\mu - ieA_\mu)\phi^*(\partial^\mu + ieA^\mu)\phi - m^2\phi^*\phi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

(a) Compute the amplitude for $\gamma(p,\epsilon) + \phi(q) \to \gamma(p',\epsilon') \to \phi(q')$, Does it vanish for $\epsilon_\mu \propto k_\mu$? Why?

b) Compute the amplitude for $\phi + \phi \to \phi + \phi$.

c) Compute the amplitude for $\phi + \phi^* \to \phi + \phi^*$.

d) Taking the nonrelativistic limit, use the results of the previous two parts to find the potential $V(\vec{r})$ between two ϕ particles, and between a ϕ and a ϕ^* particle. State which is attractive and which is repulsive.

e) Compute the amplitude and differential cross section for $\phi + \phi^* \to \gamma + \gamma$, summing over the final polarizations. (There are 3 diagrams, including the seagull one).

3. Consider the theory

$$\mathcal{L} = \bar{\psi} i \partial\!\!\!/ \psi + \tfrac{1}{2}(\partial\phi)^2 - \tfrac{1}{2}M^2\phi^2 + g\phi\bar{\psi}\psi.$$

Consider the amplitude for $\psi + \psi \to \psi + \psi$ in the limit where all momenta are small compared with M, doing an expansion in powers of $1/M$.

a) Show that the amplitude agrees with what one would compute from a theory without the scalar ϕ, but with instead

$$\mathcal{L}_{eff} = \bar{\psi} i \partial\!\!\!/ \psi + a(\bar{\psi}\psi)^2 + b\bar{\psi}\psi\partial^2(\bar{\psi}\psi) + \mathcal{O}(1/M^6).$$

Find the coefficients a and b.

b) Show that the $\mathcal{L}_{eff}$ of the previous part agrees with what would be obtained by solving the classical equations of motion for ϕ (in terms of $\bar{\psi}\psi$) in an expansion in powers of $1/M$ and then plugging that back into the original Lagrangian.