11/9 Lecture outline

• Effusion out of a hole in a box. Replace

$$f(v_z) \to \tilde{f}(z) = \begin{cases} const.p(v_z)v_z & \text{for } v_z > 0 \\ 0 & \text{for } v_z < 0 \end{cases}.$$

Gives

$$\begin{aligned}\overline{v_z^2} &= \int_0^\infty v_z^2 \exp(-\tfrac{1}{2}mv_z^2/kT)v_z dv_z / \int_0^\infty \exp(-\tfrac{1}{2}mv_z^2/kT)v_z dv_z \\ &= \tfrac{1}{2}(2kT/m)^2/\tfrac{1}{2}(2kT/m) = 2kT/m\end{aligned},$$

(using $\int_0^\infty \exp(-ax^2)x^n dx = \frac{1}{2}a^{-(n+1)/2}\Gamma(\frac{1}{2}(n+1))$), vs. $\overline{v_x^2} = \overline{v_y^2} = kT/m$. Doesn't satisfy equi-partition, and now $\overline{\epsilon} = 2kT$ - it's increased. Eventually recover equipartition, thanks to interactions. Equipartition is the most likely state. Onward to statistics!

• Binomial distribution: event with 2 possible outcomes, #1 with probability p and #2 with probability q. E.g. coin tossing, where $p = q = \frac{1}{2}$ if the coin is unbiased. Another standard example: random walk. Consider N such events, and write $N = N_1 + N_2$ with N_1 the number of them with outcome #1 and N_2 that with outcome #2. The probability that $N = N_1 + N_2$ for a given choice of N_1 (and corresponding $N_2 = N - N_1$) is

$$p(N_1) = \begin{pmatrix} N \\ N_1 \end{pmatrix} p^{N_1} q^{N_2},$$

where : $\begin{pmatrix} N \\ N_1 \end{pmatrix} \equiv N!/N_1!(N-N_1)!$ are the binomial coefficients, which enter e.g. in

$$(p+q)^N = \sum_{N_1=0}^{N} \begin{pmatrix} N \\ N_1 \end{pmatrix} p^{N_1} q^{N-N_1}.$$

Indeed, this condition shows that the probabilities are correctly normalized:

$$\sum_{N_1=0}^{N} p(N_1) = 1.$$

We can also compute

$$\overline{N_1} = \sum_{N_1=0}^{N} N_1 p(N_1) = p\frac{\partial}{\partial p}(p+q)^N = Np$$

and

$$\overline{N_1^2} = \sum_{N_1=0}^{N} N_1^2 p(N_1) = \left(p\frac{\partial}{\partial p}\right)^2 (p+q)^N = (\overline{N_1})^2 + Npq.$$

So $\overline{(\Delta N_1)^2} = Npq$. I.e. $(\Delta N_1)_{RMS} = \sqrt{Npq}$, and $(\Delta N_1)_{RMS}/\overline{N}_1 = \sqrt{\frac{q}{p}}\frac{1}{\sqrt{N}}$. Distribution is very sharply peaked around $\overline{N}_1$ for large N.