

$$\text{get } \int \frac{\vec{E}^2}{8\pi} d^3x = \sum_{a,b} \frac{q_a q_b}{2|\vec{x}_a - \vec{x}_b|}$$

should take $a \neq b$ to avoid getting ∞ .

e.g. for 2 particles finite pot = $q_1 q_2 / |\vec{x}_1 - \vec{x}_2|$

Usual electrostatic energy. ✓

Part II : solving more complicated cases

Jackson sometimes uses MKSA units.

Write $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$ ← not $\frac{\vec{v}}{c}$ as before

note now $\vec{E} \nparallel \vec{B}$ no longer have same units - bad for relativity, generally the transformations of $\vec{E} \nparallel \vec{B}$ via relativity look more complicated in MKSA units.

Compared to before, scale $q_B \rightarrow k_Q q_{\text{MKS}}$

$$\vec{E}_g \rightarrow k_Q^{-1} \vec{E}_{\text{MKS}} \quad \vec{B}_g \rightarrow c k_Q^{-1} \vec{B}_{\text{MKS}}$$

$$\text{old } \nabla \cdot \vec{E}_g = 4\pi\rho \Rightarrow k_Q^{-1} \nabla \cdot \vec{E}_{\text{MKS}} = 4\pi k_Q \rho_{\text{MKS}}$$

$$\text{Old } \nabla \times \vec{B}_g - \frac{1}{c} \frac{\partial \vec{E}_g}{\partial t} = \frac{4\pi}{c} \vec{J}_g$$

$$\Rightarrow \epsilon k_0^{-1} \nabla \times \vec{B}_{mks} - \frac{1}{c} k_0^{-1} \frac{\partial \vec{E}_{mks}}{\partial t} = \frac{4\pi}{c} k_0 \vec{J}_{mks}$$

$$\text{Old } \nabla \times \vec{E}_g + \frac{1}{c} \frac{\partial \vec{B}_g}{\partial t} = 0 \Rightarrow \nabla \times \vec{E}_{mks} + \frac{\partial \vec{B}_{mks}}{\partial t} = 0$$

Take $k_0^2 = 1/4\pi\epsilon_0$

$$\Rightarrow \nabla \cdot \vec{E}_{mks} = \rho/\epsilon_0 \quad \nabla \cdot \vec{B}_{mks} = 0$$

$$\mu_0^{-1} \nabla \times \vec{B}_{mks} - \epsilon_0 \frac{\partial \vec{E}_{mks}}{\partial t} = \vec{J}_{mks}, \quad \mu_0^{-1} \equiv c^2 \epsilon_0$$

$$\nabla \times \vec{E}_{mks} + \frac{\partial \vec{B}_{mks}}{\partial t} = 0$$

In Macroscopic media get:

$$\begin{aligned} \nabla \cdot \vec{D} &= \rho & \nabla \cdot \vec{B} &= 0 \\ \nabla \times \vec{H} - \frac{\partial \vec{D}}{\partial t} &= \vec{J} & \nabla \times \vec{E} + \frac{\partial \vec{B}}{\partial t} &= 0 \end{aligned}$$

in vacuum $\vec{D} = \epsilon_0 \vec{E} \quad \vec{B} = \mu_0 \vec{H}$

More generally $\vec{E} = \langle \vec{E}_{micro} \rangle \quad \vec{B} = \langle \vec{B}_{micro} \rangle$

10-702
42-301
42-302
42-303
42-304
42-305
42-306
42-307
42-308
42-309
42-310
42-311
42-312
42-313
42-314
42-315
42-316
42-317
42-318
42-319
42-320
42-321
42-322
42-323
42-324
42-325
42-326
42-327
42-328
42-329
42-330
42-331
42-332
42-333
42-334
42-335
42-336
42-337
42-338
42-339
42-340
42-341
42-342
42-343
42-344
42-345
42-346
42-347
42-348
42-349
42-350
42-351
42-352
42-353
42-354
42-355
42-356
42-357
42-358
42-359
42-360
42-361
42-362
42-363
42-364
42-365
42-366
42-367
42-368
42-369
42-370
42-371
42-372
42-373
42-374
42-375
42-376
42-377
42-378
42-379
42-380
42-381
42-382
42-383
42-384
42-385
42-386
42-387
42-388
42-389
42-390
42-391
42-392
42-393
42-394
42-395
42-396
42-397
42-398
42-399
42-400
42-401
42-402
42-403
42-404
42-405
42-406
42-407
42-408
42-409
42-410
42-411
42-412
42-413
42-414
42-415
42-416
42-417
42-418
42-419
42-420
42-421
42-422
42-423
42-424
42-425
42-426
42-427
42-428
42-429
42-430
42-431
42-432
42-433
42-434
42-435
42-436
42-437
42-438
42-439
42-440
42-441
42-442
42-443
42-444
42-445
42-446
42-447
42-448
42-449
42-450
42-451
42-452
42-453
42-454
42-455
42-456
42-457
42-458
42-459
42-460
42-461
42-462
42-463
42-464
42-465
42-466
42-467
42-468
42-469
42-470
42-471
42-472
42-473
42-474
42-475
42-476
42-477
42-478
42-479
42-480
42-481
42-482
42-483
42-484
42-485
42-486
42-487
42-488
42-489
42-490
42-491
42-492
42-493
42-494
42-495
42-496
42-497
42-498
42-499
42-500
42-501
42-502
42-503
42-504
42-505
42-506
42-507
42-508
42-509
42-510
42-511
42-512
42-513
42-514
42-515
42-516
42-517
42-518
42-519
42-520
42-521
42-522
42-523
42-524
42-525
42-526
42-527
42-528
42-529
42-530
42-531
42-532
42-533
42-534
42-535
42-536
42-537
42-538
42-539
42-540
42-541
42-542
42-543
42-544
42-545
42-546
42-547
42-548
42-549
42-550
42-551
42-552
42-553
42-554
42-555
42-556
42-557
42-558
42-559
42-560
42-561
42-562
42-563
42-564
42-565
42-566
42-567
42-568
42-569
42-570
42-571
42-572
42-573
42-574
42-575
42-576
42-577
42-578
42-579
42-580
42-581
42-582
42-583
42-584
42-585
42-586
42-587
42-588
42-589
42-590
42-591
42-592
42-593
42-594
42-595
42-596
42-597
42-598
42-599
42-600
42-601
42-602
42-603
42-604
42-605
42-606
42-607
42-608
42-609
42-610
42-611
42-612
42-613
42-614
42-615
42-616
42-617
42-618
42-619
42-620
42-621
42-622
42-623
42-624
42-625
42-626
42-627
42-628
42-629
42-630
42-631
42-632
42-633
42-634
42-635
42-636
42-637
42-638
42-639
42-640
42-641
42-642
42-643
42-644
42-645
42-646
42-647
42-648
42-649
42-650
42-651
42-652
42-653
42-654
42-655
42-656
42-657
42-658
42-659
42-660
42-661
42-662
42-663
42-664
42-665
42-666
42-667
42-668
42-669
42-670
42-671
42-672
42-673
42-674
42-675
42-676
42-677
42-678
42-679
42-680
42-681
42-682
42-683
42-684
42-685
42-686
42-687
42-688
42-689
42-690
42-691
42-692
42-693
42-694
42-695
42-696
42-697
42-698
42-699
42-700
42-701
42-702
42-703
42-704
42-705
42-706
42-707
42-708
42-709
42-710
42-711
42-712
42-713
42-714
42-715
42-716
42-717
42-718
42-719
42-720
42-721
42-722
42-723
42-724
42-725
42-726
42-727
42-728
42-729
42-730
42-731
42-732
42-733
42-734
42-735
42-736
42-737
42-738
42-739
42-740
42-741
42-742
42-743
42-744
42-745
42-746
42-747
42-748
42-749
42-750
42-751
42-752
42-753
42-754
42-755
42-756
42-757
42-758
42-759
42-760
42-761
42-762
42-763
42-764
42-765
42-766
42-767
42-768
42-769
42-770
42-771
42-772
42-773
42-774
42-775
42-776
42-777
42-778
42-779
42-780
42-781
42-782
42-783
42-784
42-785
42-786
42-787
42-788
42-789
42-790
42-791
42-792
42-793
42-794
42-795
42-796
42-797
42-798
42-799
42-800
42-801
42-802
42-803
42-804
42-805
42-806
42-807
42-808
42-809
42-810
42-811
42-812
42-813
42-814
42-815
42-816
42-817
42-818
42-819
42-820
42-821
42-822
42-823
42-824
42-825
42-826
42-827
42-828
42-829
42-830
42-831
42-832
42-833
42-834
42-835
42-836
42-837
42-838
42-839
42-840
42-841
42-842
42-843
42-844
42-845
42-846
42-847
42-848
42-849
42-850
42-851
42-852
42-853
42-854
42-855
42-856
42-857
42-858
42-859
42-860
42-861
42-862
42-863
42-864
42-865
42-866
42-867
42-868
42-869
42-870
42-871
42-872
42-873
42-874
42-875
42-876
42-877
42-878
42-879
42-880
42-881
42-882
42-883
42-884
42-885
42-886
42-887
42-888
42-889
42-890
42-891
42-892
42-893
42-894
42-895
42-896
42-897
42-898
42-899
42-900
42-901
42-902
42-903
42-904
42-905
42-906
42-907
42-908
42-909
42-910
42-911
42-912
42-913
42-914
42-915
42-916
42-917
42-918
42-919
42-920
42-921
42-922
42-923
42-924
42-925
42-926
42-927
42-928
42-929
42-930
42-931
42-932
42-933
42-934
42-935
42-936
42-937
42-938
42-939
42-940
42-941
42-942
42-943
42-944
42-945
42-946
42-947
42-948
42-949
42-950
42-951
42-952
42-953
42-954
42-955
42-956
42-957
42-958
42-959
42-960
42-961
42-962
42-963
42-964
42-965
42-966
42-967
42-968
42-969
42-970
42-971
42-972
42-973
42-974
42-975
42-976
42-977
42-978
42-979
42-980
42-981
42-982
42-983
42-984
42-985
42-986
42-987
42-988
42-989
42-990
42-991
42-992
42-993
42-994
42-995
42-996
42-997
42-998
42-999
43-000

National Brand

But $\vec{D} = \vec{D}[\vec{E}, \vec{B}]$
 $\vec{H} = \vec{H}[\vec{E}, \vec{B}]$ } complicated & possibly
 history dep.
 (hysteresis)

$\frac{\partial D_i}{\partial E_j} = \epsilon_{ij}$ electric permittivity or dielectric tensor

$\frac{\partial B_i}{\partial H_j} = \mu_{ij}$ magnetic permeability tensor

$\epsilon_{ij}[\vec{E}, \vec{B}]$ $\mu_{ij}[\vec{E}, \vec{B}]$ generally complicated

Electrostatic system of stationary charges

point charges q_i @ positions \vec{x}_i

$$\vec{E}(\vec{x}) = \frac{1}{4\pi\epsilon_0} \sum_i q_i \frac{(\vec{x} - \vec{x}_i)}{|\vec{x} - \vec{x}_i|^3} \quad \text{superposition}$$

$$\vec{E} = -\nabla\phi, \quad \phi(\vec{x}) = \frac{1}{4\pi\epsilon_0} \sum_i \frac{q_i}{|\vec{x} - \vec{x}_i|}$$

$$\rho(\vec{x}) = \sum_i q_i \delta^3(\vec{x} - \vec{x}_i)$$

$$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0} \Rightarrow -\nabla^2 \phi = \rho / \epsilon_0$$

$$\text{so } \nabla^2 \left(\frac{1}{|\vec{x} - \vec{x}_i|} \right) = -4\pi \delta^3(\vec{x} - \vec{x}_i)$$

Many ways to show (e.g. Fourier transform)

here just note in spherical coords

$$\nabla^2 \psi = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial \psi}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial \psi}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 \psi}{\partial \phi^2}$$

so formally $\nabla^2 \left(\frac{1}{r} \right) = 0$, valid

everywhere except at $r=0$. Gauss law

$$\int_V \nabla^2 \left(\frac{1}{r} \right) = \int_S \vec{\nabla} \cdot \left(\frac{1}{r} \right) \cdot d\vec{S} = \int_S -\frac{1}{r^2} \hat{r} \cdot d\vec{S}$$

$$d\vec{S} = \hat{r} r^2 \underbrace{d(\cos \theta) d\phi}_{d\Omega} \quad \text{so } \int_V \nabla^2 \left(\frac{1}{r} \right) = -\int_S d\Omega$$

$= -4\pi$ if $r=0$ inside S or zero otherwise

$$\nabla^2 \left(\frac{-1}{4\pi |\vec{x} - \vec{x}_i|} \right) = \delta^3(\vec{x} - \vec{x}_i)$$

↑ "Green function of Laplacian ∇^2 "

Solve $\nabla^2 \phi = -\rho/\epsilon_0$ ← via "Poisson Eqn"

$$\phi(\vec{x}) = \int d^3\vec{y} \frac{1}{4\pi\epsilon_0} \frac{\rho(\vec{y})}{|\vec{x} - \vec{y}|}$$

$$\begin{aligned} \nabla^2 \phi(\vec{x}) &= \int d^3\vec{y} \frac{1}{4\pi\epsilon_0} \rho(\vec{y}) (-4\pi \delta^3(\vec{x} - \vec{y})) \\ &= -\rho(\vec{x})/\epsilon_0 \quad \checkmark \end{aligned}$$

Plug in $\rho(\vec{y}) = \sum_i q_i \delta^3(\vec{y} - \vec{x}_i)$

get $\phi(\vec{x}) = \sum_i \frac{q_i}{4\pi\epsilon_0} \frac{1}{|\vec{x} - \vec{x}_i|}$ as before

More general Green function:

$$\nabla_x^2 G(\vec{x}, \vec{y}) = -4\pi \delta^3(\vec{x} - \vec{y})$$

$$G(\vec{x}, \vec{y}) = \frac{1}{|\vec{x} - \vec{y}|} + F(\vec{x}, \vec{y})$$

$$\nabla^2 \Phi(\vec{x}, \vec{y}) = 0 \quad \text{"Laplace eqn"}$$

$$\phi(\vec{x}) = \frac{1}{4\pi\epsilon_0} \int_V d^3\vec{y} \ G(\vec{x}, \vec{y}) \rho(\vec{y})$$

$$+ \frac{1}{4\pi} \oint_S \left[G(\vec{x}, \vec{y}') \nabla' \phi - \phi(\vec{x}') \nabla' G(\vec{x}, \vec{x}') \right] \cdot \hat{n} \, dA'$$

Obtain from math identity

$$\int_V (\phi \nabla^2 \psi - \psi \nabla^2 \phi) \, d^3x = \oint_S (\phi \nabla \psi - \psi \nabla \phi) \cdot d\vec{A}$$

for $\psi = \frac{1}{4\pi\epsilon_0} G$.

Specify $F \neq \therefore G$ by choice of BCs eg

"Dirichlet" specify $\Phi|_{\text{bdy}}$

"Neumann" " $\vec{n} \cdot \nabla \Phi|_{\text{bdy}}$

e.g. Dirichlet $\Rightarrow G(\vec{x}, \vec{x}') = 0$ for \vec{x}' on bndy

Electrostatic energy: in CGS we had $H_{\text{field}} = \int \frac{d^3x}{8\pi} (\vec{E}^2 + \vec{B}^2)$

now $\vec{B} = 0$ here $\frac{\vec{E}_{\text{CGS}}^2}{8\pi} \Rightarrow \frac{\epsilon_0}{2} \vec{E}_{\text{MKS}}^2$

so $H_{\text{field}} = \frac{\epsilon_0}{2} \int d^3x \vec{E}^2$ (for $\vec{B} = 0$)

$\vec{E} = -\nabla\phi \hookrightarrow H_{\text{field}} = \frac{\epsilon_0}{2} \int d^3x \nabla\phi \cdot \nabla\phi$

$H_{\text{field}} = \frac{\epsilon_0}{2} \int d^3x \left[\nabla(\phi \nabla\phi) - \phi \nabla^2\phi \right]$

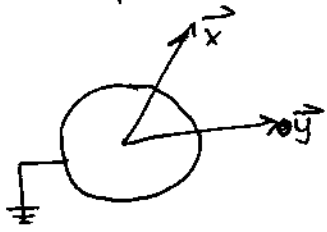
$= \frac{\epsilon_0}{2} \oint_S \phi \nabla\phi \cdot d\vec{S} - \frac{\epsilon_0}{2} \int d^3x \phi (-\rho(\vec{x})/\epsilon_0)$

$= \downarrow$ surface contrib $+ \frac{1}{8\pi\epsilon_0} \iint \frac{\rho(\vec{x})\rho(\vec{x}')}{|\vec{x}-\vec{x}'|} d^3x d^3x'$

must exclude self energy at $\vec{x} = \vec{x}'$ to get finite answer. E.g. $\rho = \sum_i q_i \delta(\vec{x} - \vec{x}_i)$

$H_{\text{field}} = \text{surface term} + \frac{1}{8\pi\epsilon_0} \sum_{i,j} \frac{q_i q_j}{|\vec{x}_i - \vec{x}_j|}$

Example of B.C. problem



Ground conducting sphere charge $q @ \vec{y}$, observer @ \vec{x}

$$\phi(\vec{x}) = \frac{q/4\pi\epsilon_0}{|\vec{x} - \vec{y}|} + \frac{q'/4\pi\epsilon_0}{|\vec{x} - \vec{y}'|} \quad \leftarrow \text{image charge contrib.}$$

$$\vec{y}' = y' \hat{n}' \quad \vec{y} = y \hat{n} \quad \vec{x} = x \hat{n}$$

$$\phi(\vec{x}) \Big|_{|\vec{x}|=a} = 0 \quad \leftarrow \text{B.C.}$$

$$\phi(a) = \frac{q/4\pi\epsilon_0}{|a\hat{n} - y\hat{n}|} + \frac{q'/4\pi\epsilon_0}{|a\hat{n} - y'\hat{n}'|} = 0$$

~~cancel terms~~

$$\frac{q/4\pi\epsilon_0}{y \left| \hat{n} - \frac{y}{a} \hat{n}' \right|} + \frac{q'/4\pi\epsilon_0}{y' \left| \hat{n}' - \frac{a}{y'} \hat{n} \right|} = 0$$

$$\text{sol'n: } \frac{q}{a} = -\frac{q'}{y'} \quad \frac{y}{a} = \frac{a}{y'}$$

$$\Rightarrow q' = -a^2 q / y \quad y' = a^2 / y$$

$$\Phi = \frac{q}{4\pi\epsilon_0} \left(\frac{1}{\sqrt{x^2 + y^2 - 2xy \cos \gamma}} + \frac{-a/y}{\sqrt{x^2 + \frac{a^4}{y^2} - \frac{2x a^2 \cos \gamma}{y}}} \right)$$

$$\Phi = \frac{q}{4\pi\epsilon_0} \left((x^2 + y^2 - 2xy \cos \gamma)^{-1/2} - \left(\frac{x^2 y^2}{a^2} + a^2 - 2xy \cos \gamma \right)^{-1/2} \right)$$

$$x = |\vec{x}| \quad y = |\vec{y}| \quad \text{here, of course}$$

Can find surface charge

$$\sigma = -\epsilon_0 \left. \frac{\partial \Phi}{\partial x} \right|_{x=a}$$

$$\sigma = \frac{+q}{4\pi} \left(a^2 + y^2 - 2ay \cos \gamma \right)^{-3/2} \left(a - y \cos \gamma \right)$$

$$- \frac{1}{a} y^2 + y \cos \gamma \Big) = -\frac{aq}{4\pi} (y^2 - a^2) (*)^{-3/2}$$

$$\text{Can verify } \oint d\Omega a^2 \sigma = -\frac{qa}{4\pi} = q' \quad \checkmark$$

To get insulated rather than ground conductor
add charge $Q - q'$ smoothly distributed

$$\Phi(\vec{x}) = \frac{1}{4\pi\epsilon_0} \left[\frac{q}{|\vec{x}-\vec{y}|} - \frac{aq}{y \left| \vec{x} - \frac{a^2}{y^2} \vec{y} \right|} + \frac{\left(Q + \frac{aq}{y} \right)}{|\vec{x}|} \right]$$

Also fixed potential @ sphere $Q \rightarrow q' \rightarrow V_a$

Consider now complicated looking problem

• observer



sphere with given potential $\phi(a, \theta, \phi)$



charge distribution ρ in space

Solve using Green function

$$G(\vec{x}, \vec{x}') = \frac{1}{|\vec{x}-\vec{x}'|} - \frac{a}{\vec{x}' \left| \vec{x} - \frac{a^2}{x'^2} \vec{x}' \right|}$$

$$= \left(x^2 + x'^2 - 2xx' \cos \gamma \right)^{-1/2} - \left(\frac{x^2 x'^2}{a^2} + a^2 - 2xx' \cos \gamma \right)^{-1/2}$$

$\gamma =$ angle between \vec{x} & \vec{x}'

$G=0$ on sphere

$$\Phi(\vec{x}) = \frac{1}{4\pi\epsilon_0} \int_V d^3x' G(\vec{x}, \vec{x}') \rho(\vec{x}') + \frac{1}{4\pi} \oint_S (G \nabla \Phi - \Phi \nabla G) \cdot d\vec{S}$$

$$\text{So } \Phi(\vec{x}) = \frac{1}{4\pi\epsilon_0} \int_V d^3x' G(\vec{x}, \vec{x}') \rho(\vec{x}')$$

$$\frac{1}{4\pi} \int d(\cos\theta') d\phi' \Phi(a, \theta', \phi') \left. \frac{\partial G}{\partial n'} \right|_{x'=a}$$

$$\frac{\partial G}{\partial n'} = - \frac{(x^2 - a^2)}{a(x^2 + a^2 - 2ax \cos\theta')^{3/2}}$$

The problem is solved (assuming we can evaluate the integral)!

Study sol's of Laplace eqn $\nabla^2 \Phi = 0$

e.g. in box $\Phi = X(x) Y(y) Z(z)$

$$\frac{1}{X} \frac{d^2 X}{dx^2} + \frac{1}{Y} \frac{d^2 Y}{dy^2} + \frac{1}{Z} \frac{d^2 Z}{dz^2} = 0$$

each must be constant

$$c_x + c_y + c_z = 0$$

Sol's = $\sum \sin \text{ or } \cosh$
 $c < 0$ $c > 0$

$$X'' = c_x X$$

$$Y'' = c_y Y$$

$$Z'' = c_z Z$$

Spherical coordinates: $\nabla^2 \Phi =$

$$\frac{1}{r} \frac{\partial^2}{\partial r^2} (r \Phi) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial \Phi}{\partial \theta} \right)$$

$$+ \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 \Phi}{\partial \phi^2}$$

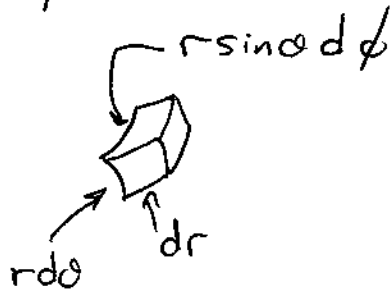
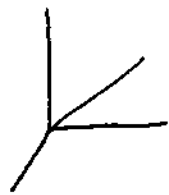
$$ds^2 = \sum_n h_n^2 d\xi_n^2$$

$$\nabla F = \sum_n \frac{\hat{n}}{h_n} \frac{\partial F}{\partial \xi_n}$$

$$\vec{\nabla} \cdot \vec{F} = \frac{1}{h_1 h_2 h_3} \left(\frac{\partial}{\partial \xi_1} (F_1 h_2 h_3) + \dots \right)$$

$$\vec{\nabla} \times \vec{F} = \frac{\hat{a}_1}{h_2 h_3} \left(\frac{\partial}{\partial \xi_2} (h_3 F_3) - \frac{\partial}{\partial \xi_3} (h_2 F_2) \right) + \dots$$

aside:



$$\nabla \Phi = \frac{\partial \Phi}{\partial r} \hat{r} + \frac{1}{r} \frac{\partial \Phi}{\partial \theta} \hat{\theta} + \frac{1}{r \sin \theta} \frac{\partial \Phi}{\partial \phi} \hat{\phi} + \dots$$

$$\nabla \cdot \vec{A} = \frac{1}{r^2 \sin \theta} \left[\frac{\partial}{\partial r} (r^2 \sin \theta A_r) + \frac{\partial}{\partial \theta} (r \sin \theta A_\theta) + \frac{\partial}{\partial \phi} (r A_\phi) \right]$$

understand via Gauss law explain

$$\vec{\nabla} \times \vec{A} = \frac{\hat{r}}{r^2 \sin \theta} \left(\frac{\partial}{\partial \theta} (r \sin \theta A_\phi) - \frac{\partial}{\partial \phi} (r A_\theta) \right)$$

$$+ \frac{\hat{\theta}}{r \sin \theta} \left(\frac{\partial}{\partial \phi} (A_r) - \frac{\partial}{\partial r} (r \sin \theta A_\phi) \right) + \frac{\hat{\phi}}{r} (\dots)$$

