1. Evaluate $F_{\mu\nu}F^{\mu\nu}$, $F_{\mu\nu}\widetilde{F}^{\mu\nu}$, and $\widetilde{F}_{\mu\nu}\widetilde{F}^{\mu\nu}$, where $\widetilde{F}^{\mu\nu} \equiv \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}$ is as given in lecture.

2. Consider the following three cases for the electric and magnetic fields in some frame K (with P some constant):

i. $\vec{E} = (4P, 0, 0)$ and $\vec{B} = (0, 5P, 0)$

ii. $\vec{E} = (5P, 0, 0)$, $\vec{B} = (0, 4P, 0)$

iii. $\vec{E} = (P, 0, 0)$, $\vec{B} = (P, 2P, 0)$.

a. In which of these cases is there a frame K' where the field is purely electric? For each such case, write out the Lorentz transformation, $x^\mu = \Lambda^\mu_\nu x'^\nu$, between the frame K and the frame K' where $\vec{E}' = E'_0\widehat{x}$ and $\vec{B}' = 0$? What is E'_0 in terms of P?

b. In which of the above cases is there a frame K' where the field is purely magnetic? For each such case, write out the Lorentz transformation $x^\mu = \Lambda^\mu_\nu x'^\nu$, between the frame K and the frame K' where $\vec{E}' = 0$ and $\vec{B}' = B'_0\widehat{y}$? What is B'_0 in terms of P?

c. For the case or cases found in part (b), solve for the motion of a charge q particle which is at $x^\mu = 0$, with velocity $\vec{v} = 0$, in frame K. Solve for the trajectory $\vec{x}\,'(t')$ seen in the frame K', where the field is purely magnetic.

3. Define $X_{\mu\nu\rho} \equiv \partial_\mu A_{\nu\rho} + \partial_\nu A_{\rho\mu} + \partial_\rho A_{\mu\nu}$, with $A_{\mu\nu} = -A_{\nu\mu}$ an antisymmetric tensor. Verify $X_{\mu\nu\rho} = -X_{\nu\mu\rho}$. How many *independent* non-zero components does the tensor $X_{\mu\nu\rho}$ have?