Approximate formula for meson masses:

\[ m(q_1q_2) \approx m_1 + m_2 + \frac{A}{m_1m_2} \langle \vec{S}_1 \cdot \vec{S}_2 \rangle. \]

\[ m_u \approx m_d \approx 0.307 \text{GeV}, \ m_s \approx 0.4900 \text{GeV}, \ A \approx 0.06 \text{GeV}^3. \] Note \( m_{q', \text{naive}} \approx 355 \text{MeV} \) vs \( m_{q', \text{actual}} \approx 958 \text{MeV}. \)

- \( j = 0 \) baryons and symmetry.
- Approximate formula for baryon masses:

\[ m(q_1q_2q_3) \approx m_1 + m_2 + m_3 + A' \left( \frac{\langle \vec{S}_1 \cdot \vec{S}_2 \rangle}{m_1m_2} + 2 - \text{perms} \right). \]

\[ m_u \approx m_d \approx 0.365 \text{GeV}, \ m_s \approx 0.540 \text{GeV}, \ A' \approx 0.026 \text{GeV}^3. \] Comments.

- Aside on magnetic moments and magnetic dipole-dipole interactions. Recall why a classical current loop has \( \vec{\mu} \propto \vec{L} \): a charge \( q \), of mass \( m \), moving in a circle of radius \( r \) with angular frequency \( \omega \) has \( \vec{L} = mw^2\hat{n} \) and \( \vec{\mu} = I \pi r^2 \hat{n} \), with current \( I = q/T = q\omega/2\pi \). So \( \vec{\mu}_{\text{classical}} = q\vec{L}/2m \). A quantum spin has \( \vec{\mu}_{\text{quantum}} = gg\vec{S}/2m \), where \( g = 2 \) for a free Dirac Fermion and quantum corrections from the interactions modify that further, e.g. for QED \( g = (1 + \alpha/2\pi + \ldots) \).

- quark model predictions for magnetic moments: \( \mu_p \approx \frac{4}{3} \mu_u - \frac{1}{3} \mu_d, \ \mu_n \approx \frac{4}{3} \mu_d - \frac{1}{3} \mu_u \), and \( \mu_u \approx -2 \mu_d \).

- Help with HW questions. Recall Clebsch Gordon coefficients, which can be found via \( T_{\pm} |I, I_3 \rangle = \sqrt{I(I + 1) - I_3(I_3 \mp 1)} |I, I_3 \pm 1 \rangle \). E.g. \( |3/2, 3/2 \rangle = |1, 1 \rangle |1/2, 1/2 \rangle \). Lower both sides to get \( |3/2, 1/2 \rangle = \sqrt{1/3} |3/2, 1 \rangle |1/2, -1/2 \rangle + \sqrt{2/3} |1, 0 \rangle |1/2, 1/2 \rangle \), etc. compare with table of CG coefficients from the PDG. Physical observables involve squaring amplitudes, which will lead to ratios that differ by squares of the CG coefficients.

- Dirac equation for \( N \) Fermions. Explain the \( U(N) \) symmetry if all have the same mass (and electric charge). (For \( m = 0 \), it is actually a \( U(N)_L \times U(N)_R \) symmetry, at least classically. More on this later.) \( SU(3) \) flavor rotates the \( (u, d, s) \) quarks. They have different charges and masses, but as far as the strong force is concerned they are all the same. This is why \( SU(3)_F \) is a pretty good, but approximate, global symmetry.

- A few generalities about \( SU(N) \) and the fundamental, anti-fundamental, and adjoint representations.

- \( SU(3) \). Recall \( |SU(N)| = N^2 - 1 \), so \( |SU(3)| = 8 \). The fundamental representation is the \( 3 \) and the anti-fundamental rep is the \( \bar{3} \). These are the analogs of spin 1/2 for \( SU(2) \);
for general $SU(N)$ the $\mathbf{N}$ and $\overline{\mathbf{N}}$ differ, but for $SU(2)$ they happen to be equivalent. For general $SU(N)$ we can think of the fundamental as acting on $v^c$ and the anti-fundamental on $\bar{v}_c$, and $SU(N)$ preserves $\delta^d_c$ and $\epsilon_{c_1...c_N}$ and $\epsilon^{c_1...c_N}$. For $SU(2)$, we can relate $\bar{v}_c = \epsilon_{cd}v^d$.

- The Gell-Mann matrices and the $\mathbf{3}$ vs the $\overline{\mathbf{3}}$.
- Illustrate the $\mathbf{3}$, $\overline{\mathbf{3}}$, and $\mathbf{3} \times \mathbf{3} = \overline{\mathbf{3}} + \mathbf{6}$ via their weights in the $(T_3,T_8)$ plane.

Next time:
- Application: approximate $SU(3)_F$ global symmetry for the $(u,d,s)$ quarks. Mesons and baryons, spectrum and numbers.